WHY MOST DEMANDS FOR MONEY EQUATIONS ARE NOT DEMANDS FOR MONEY. HOMOGENEITY IS NO LONGER THE ANSWER.

POR QUÉ LA MAYORÍA DE LAS ECUACIONES DE DEMANDA POR DINERO NO SON DEMANDAS POR DINERO. LA HOMOGENEIDAD YA NO ES MÁS LA RESPUESTA.

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ABSTRACT

Homogeneity of degree zero concerning the "general price level" and income used to be an important first filter to judge whether a demand for money specification was valid. Since this filter was overcome by the literature, monetary economics has lacked a similar criterion. In this paper we posit one: for a demand-for-money equation to be considered as such, its inverse needs to be able to represent "aggregate supply"; we defend the validity of this criterion and show that most demand-for-money specifications do not fulfill it. Also, this criterion leads us to prove mathematically a Mises's Regression Theorem-like proposition —we test our result empirically using the U.S. price level since 1800. We center our discussion first in the Cambridge equation, to fix ideas, and we then generalize our arguments.

Key words: demand for money; Mises's Regression Theorem; price level; quantity theory

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RESUMEN

La homogeneidad de grado cero con respecto al "nivel general de precios" y al ingreso solía ser un importante primer filtro para juzgar si una especificación de demanda por dinero era válida. Desde que la literatura superó este filtro, la economía monetaria ha carecido de un criterio similar. En este artículo proponemos uno: para que una ecuación de demanda por dinero sea considerada como tal, su inversa debe poder representar la "oferta agregada"; defendemos la validez de este criterio y demostramos que la mayoría de las especificaciones de demanda por dinero no lo satisfacen. También, este criterio nos lleva a probar matemáticamente un resultado similar al Teorema de Regresión de Mises —evaluamos empíricamente nuestro resultado usando el nivel de precios estadounidense desde 1800. Centramos nuestra discusión primero en la ecuación de Cambridge, para ordenar las ideas, y luego generalizamos nuestros argumentos.

Palabras claves: demanda por dinero; Teorema de Regresión de Mises; nivel de precios; teoría cuantitativa

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1. INTRODUCTION

Homogeneity of degree zero in prices and income is a fundamental characteristic of (Walrasian) demand functions. Hence, the problem of homogeneity of the demand for money concerning the "general price level" was a central issue in monetary theory (Goldfeld and Sichel 1990). So, the homogeneity of the Cambridge equation used to be one of the main challenges for quantity theorists.

However, this "filter" criterion, to determine whether a specification represents a valid demand for money, has been mostly overcome in the literature, even by the Cambridge equation. It has been suggested that Von Mises (1953) solved this problem because what he calls the objective value of money is practically the same as the inverse of the "general price level" (Milei and Giacomini 2017); but this seems contradictory with other interpretations that suggest Mises' (1953) correction of the quantity equation leads to an expression that is not even a (Walrasian) demand for money: not dependent on income but on the level of transactions, but still in a Cambridge equation representation[1] (Evans and Thorpe 2013).

However, it has already been shown that a homogeneous traditional Cambridge equation can be derived from Cobb-Douglas preferences, defending very different notions of which variable should be the price of money[2] (Monge 2021). The most common solution in the literature, even adopted by the manuals (Galí 2008), has been to incorporate interest rates to have additional prices for which to get the desired homogeneity.

All of the above leads us to our current situation: having "surpassed" the homogeneity issue, monetary economics was left without any other main criterion to judge if a function is a correct specification for demand for money. In this essay, we propose such a criterion, defend its validity, and show that most demand-for-money functions cannot fulfill it: for a demand-for-money equation to be considered as such, its inverse needs to be able to represent "aggregate supply".

2. A NEW CRITERION

As Rothbard (1970) points out, the supply of goods is the inverse of the demand for money: selling commodities for money is analogous to purchasing money (with commodities)[3]. So, if the highly aggregative context of the Cambridge equation is appropriate, if we invert the (aggregate) demand for money, we should get the "aggregate supply" function. However, with the Cambridge equation, this is not the case:

$$M_t = kP_tY_t \Leftrightarrow Y_t = \frac{M_t}{kP_t} \quad (1)$$

As usual, M represents the money supply, P represents the "general price level", Y represents "aggregate output", and k represents the fraction of income that households use to demand money —subindex t represents the corresponding variable in the "t-th" period.

1. This interpretation matches the argument we want to defend in this paper: that most demand for money specifications are reduced form equations.

2. The result is derived from a static model; deriving the Cambridge equation from this kind of environment should warn us about the lack of dynamics we are introducing when considering it as a structural equation.

3. In fact, this idea can be traced back to Patinkin (1949), but we decided to quote Rothbard (1970) as our source because of rigor: Patinkin (1949) considers demand for money and supply of goods to be identical — Rothbard (1970) advanced the argument to say that the latter is the inverse of the first.

Therefore, equation (1) tells us that "aggregate supply" quantities are negatively related to the "general price level", which is a contradiction.

Note also that this result exposes a problem about the analytical instrument: the equation of exchange employed as a structural demand for money representation. The assumptions of the quantity theory are not even discussed; if there is an Austrian analytical setting that can be compatible with some of the quantity theory assumptions, it would be Rothbard's (1970): since he postulates that all supply curves are perfectly inelastic. And we still get a contradiction because the instrument alone is inadequate. This reasoning leads us to posit and defend a new criterion to evaluate whether a demand for money specification is valid: for a demand for money equation to be considered as such, its inverse needs to be able to represent "aggregate supply"[4]. And the Cambridge equation does not pass this filter. However, we can extend the result to arbitrary demand for money functions[5] with only three axioms:

- A1. Money is a normal good.
- A2. The inverse of the "general price level" is the price of money.
- A3. Money demand is differentiable in equilibrium concerning the "general price level" and "aggregate income".

Now, if the money market is in equilibrium in the "t-th" period, the money supply function, M_t^O, and the money $M_t^D(Y_t^D, P_t, \cdot)$ demand, , functions satisfy:

$$M_t^O(\bullet) = M_t^D(Y_t, P_t, \bullet) \quad (2)$$

Since (2) is valid for any monetary system, it is valid for a system in which the money supply is exogenous and then $(M_t^O \cdot) = M_t^O \in \mathbb{R}^+$. So, we can define

$$G(Y_t, P_t) \equiv M_t^O - M_t^D(Y_t, P_t, \bullet) = 0 \quad (3)$$

Applying the Implicit Function Theorem

$$\frac{\partial Y_t}{\partial P_t} = -\frac{G_{P_t}}{G_{Y_t}} = -\frac{\frac{\partial M_t^D}{\partial P_t}}{\frac{\partial M_t^D}{\partial Y_t}} \quad (4)$$

Because of A.1, $\frac{\partial M_t^D}{\partial Y_t} > 0$; and because of A.1 together with A.2, $\frac{\partial M_t^D}{\partial P_t} > 0$—Giffen goods are always inferior goods.
Hence, from (4), $\frac{\partial Y_t}{\partial P_t} < 0$; implying that inverting $M_t^D(Y_t, P_t, \cdot)$ leads to a relationship between prices and production that cannot correspond to the "aggregate supply" function. Note that it is irrelevant for our proposition whether the interest rate is included or not in the function[6].

3. MISES'S REGRESSION THEOREM

Our results in the previous section have a Mises's Regression Theorem-like proposition as their corollary. According to (4), if we do not explicitly model expectations (i.e., if we do not incorporate future values of P_t as arguments of the demand for money function), the only way in which a demand for money can incorporate the "general price level" is by incorporating its lags, since we cannot include P_t together with our axioms[7]. Now, if we axiomatically accept that the "general price level" depends on the demand for money, the following is a valid condition:

$$P_t = f(P_{t-1}) \quad (5)$$

4. Failing to pass this filter is not only harmful in terms of having an inappropriate demand for money representation; for example, all arguments in Potuzak (2016) about Hayek's MV-rule rely heavily on considering that the relationship between P and Y in the equation represents the elasticity of "aggregate demand".

5. So, keeping the other huge quantity theory assumption (constant k) does not alter the result we find in (1).

6. It is irrelevant for our proposition even whether the interest rate is considered as the price of money or not (instead of just as the main opportunity cost) if we substitute A.2. simply assuming from the beginning that $\frac{\partial M_t^D}{\partial Y_t} > 0$.

7. Money demand does not depend on the current price level, so neither the aggregate supply: so aggregate supply is perfectly inelastic as Rothbard (1970) suggests!

And basic results in difference equations would imply that there exists a function $F(\cdot)$ such that $P_t=F(P_0)$; i.e., the value of money can be traced back to its "original" value. Note that we use only one lag without loss of generality: mathematically, with more lags our solution to P_t still depends on the initial value, P_0; economically, see Huerta de Soto (2005) for a discussion about the subjectivity of praxeological time.

Also, without imposing any axiom at all, if the economy uses non-fiat money, we can write money supply as $M_t^O(P_t)$, and solving for P_t from (2) gives an expression like (5) and P_0 is directly interpretable: the value of money as a commodity (only as a commodity, not money). Even more, if the economy uses non-fiat money, as Evans and Thorpe (2013) posit, stable prices lead to $E_t[P_{t+j}]= P_{t-1}$: hence, (5) is valid in general (i.e., even when we consider expectations) for this kind of environments.

In conclusion, we can make the following statement: mathematically, it can be shown that, without explicitly modeling expectations, Mises's Regression Theorem is true for non-fiat money and is (highly) probable to be true for fiat money —and is (highly) probable to be true, even explicitly modeling expectations, for non-fiat money. This conclusion could be interpreted as evidence in favor of the view exposed in Davidson and Block (2015), according to which Mises's Regression Theorem is relevant only when the medium of exchange arises out of barter.

We can test empirically our theoretical findings about Mises's Regression Theorem. Equation (5) suggests that a growing "general price level" must follow a unit root process, not a trend-stationary process. We can evaluate this hypothesis with the famous Kwiatkowski, Phillips, Schmidt, and Shin (1992), or KPSS, test using the consumer price index (CPI) of the United States from 1800 to 2022.

Table 1: KPSS test for the average annual CPI of the United States (1800-2022).

Test statistic		
1800-2022	1800-1970	1971-2022
0.91619***	0.6831***	0.10795

Source: own elaboration.
a/ * means "significant at the 10%", ** means "significant at the 5%", and *** means "significant at the 1%"
b/ Maximum lag orders were taken from Schwert's (1989) criterion.

As can be seen from Table 1, we can reject the null hypothesis of trend-stationarity for the whole sample and for the period that goes from the beginning of the sample to the last year before the definitive cessation of the gold standard, i.e., 1971. However, we cannot reject the null hypothesis (at any conventional significance level) for the period that goes from the definitive cessation of the gold standard to the end of the sample[8].

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8. These results must be interpreted with caution: we are presenting (very) long run trends; from 1800 to 1971 the gold standard suffered multiple transformations (even cessations). Of course, in addition to the caution that one must have due to the criticism that Mises' theorem has received, for example in Rallo (2023).

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